Chelsea and Eliza enjoy learning Addition

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Chelsea and her sister Eliza were starting their first day of kindergarten with their teacher, Mrs. Matisse who was named after a famous French artist. What they did not know was that their grandfather, the famous Math Magician, was invited to participate in the class to try out a new approach in teaching mathematics. Mrs. Matisse was going to teach them lots of things: how to count forward and backward, how to skip count, the days of the week, the months of the year, how to have good manners and so much more. Their grandfather was amazed at what Mrs. Matisse was going to be able to do with all these students. He thought that she must be the world's best teacher.

To add to the multitude of wondrous things she was going to teach, he suggested that the students should be taught the five positive pairs of integer complements of 10. He wrote the following 5 numbers:

9 8 7 6 5

The underneath these numbers, Mrs. Matisse wrote:

1 2 3 4 5

She then told her students that each of these pairs added to ten. She got out her bead board and showed them the following:
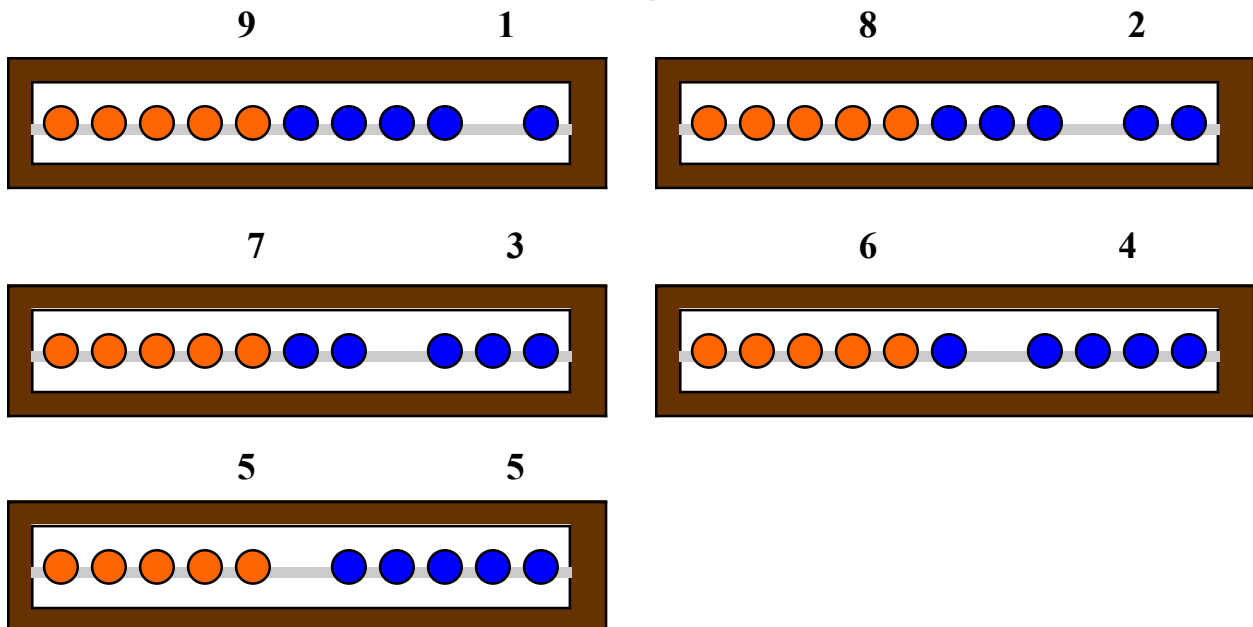
Chelsea noticed that as Mrs. Matisse moved the beads to the right, she was creating the number pairs (ten complements) that she and the Math Magician had writ-

ten on the board. Eliza realized that they all added to ten since Mrs. Matisse neither took beads off or put beads on the wire.

Then Mrs. Matisse played a game with them in which she would say a number and the students would tell her the tens complement. Thus when she said 9, they said 1, and when she said 3 they said 7. Little by little the class was getting better and better at giving the right response because they were starting to memorize the facts To make it easier for them, she just used the following set:

9   8   5
1   2   5

When they memorized these, she gave them the rest:

7   6
3   4

Since the students already knew how to count backwards, she decided to play a game that they might like. First, she wrote down the following set of numbers, and then they had to tell her what numbers went under them:

9	**2**	**8**	**3**	**7**	**4**	**6**	**5**	Mrs Matisse wrote these
1	8	2	4	3	6	4	5	The students said this

Mrs. Matisse told Chelsea, Eliza and the rest of the class that they had done very well. She said that she wanted to do more by telling her the number which was one less then the dark numbers on the top. So for 9, they said 8; and for 4, they said 3. When the students completed the task, they had the following:

9	**2**	**8**	**3**	**7**	**4**	**6**	**5**	Mrs Matisse wrote these
81	**1**8	**7**2	**2**4	**6**3	**3**6	**5**4	**4**5	The students said the dark numbers

Eliza had been taught by her grandfather how to multiply by 9. Eliza said in delight, why Mrs. Matisse, you showed us a clever way to multiply by nine. Chelsea responded that it was a good thing that they had been taught to count backwards. Little did this kindergarten class know that they knew their nine facts for multiplication better than most third graders did. They also did not now that the tens complements skill would help then add large columns of numbers and make it easier to learn subtraction.

Mrs. Matisse looked at the Math Magician, and asked if he knew any other clever tricks. "Yes", he said, "let me show you." He took the bead board and moved the beads as follows:

5 + 2 = 7   **5 + 1=6**
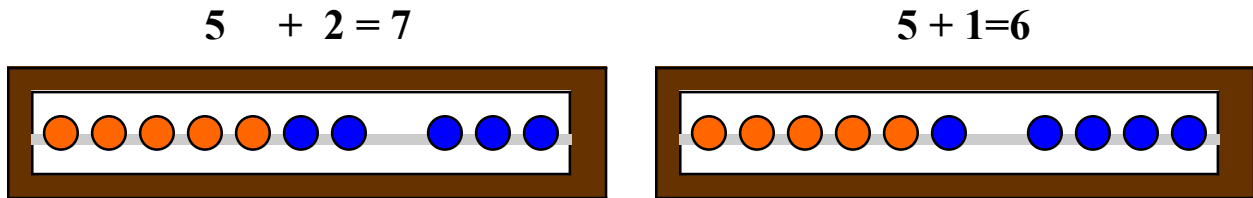
Then, Mrs. Matisse had the children memorize that 5+2=7 and 5+1=6. Now she pulled out a double bead board called a Rekenrek.
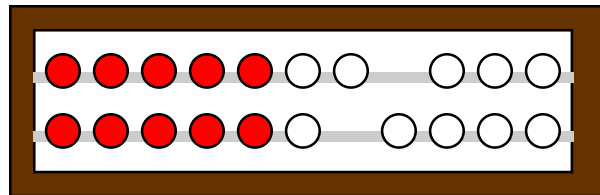
She showed that the seven was five red beads and two white beads, and that the six was five red beads and white bead. From the tens complements, Chelsea and Eliza knew that five plus five was ten. Eliza piped up that two plus one was three. Why ten and three is thirteen, so 7+6 must equal thirteen.

Then Mrs. Matisse asked Eliza to explain this because she had not yet taught the class how to add ten to a number. To help Eliza to explain this, Mrs. Matisse wrote the following on the board:

$$
\begin{array}{ccccccc}
2 & 3 & 4 & 5 & 7 & 8 & 9 \\
+10 & +10 & +10 & +10 & +10 & +10 & +10 \\
\hline
12 & 13 & 14 & 15 & 17 & 18 & 19
\end{array}
$$

Eliza said that you copy the number and put a one in front of it. She said that for 10 plus 8, this became 18 which she pronounce eight ten. Mrs. Matisse laughed and said the ten was pronounced teen, so it became eighteen.

This had been a busy few weeks for the class. Mrs. Matisse said that she and the class had enjoyed the math tricks so much that they wanted to learn more. The Math Magician said that he had another trick up his sleeve and brought in a magic square board from his math museum—Math and Physics Exploration.

The board and numbers looked like this:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
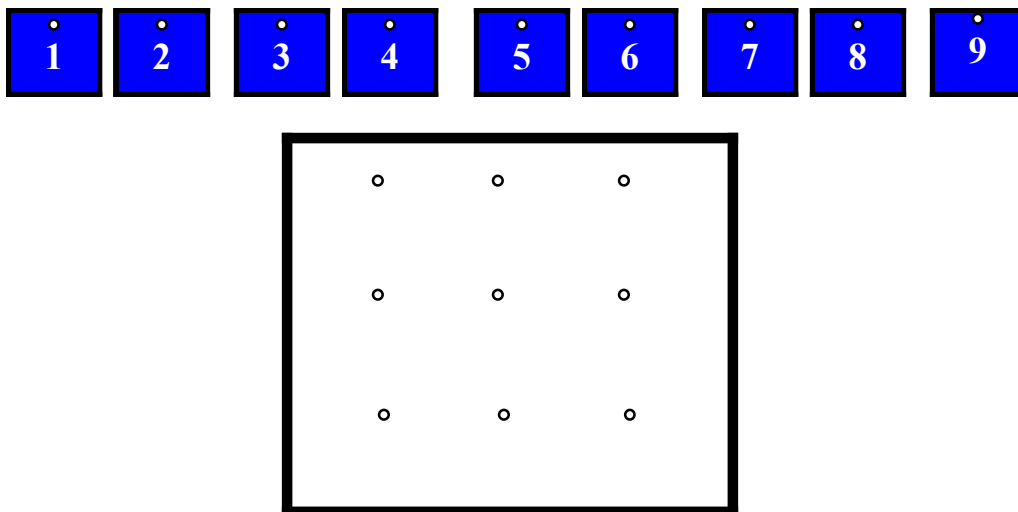
The Math Magician new that Mrs. Matisse had taught her students how to skip count. Thus, he asked one of the students to put the even numbers in the corners. Eliza volunteered to do this:

| 1 | 3 | 5 | 7 | 9 |

2   4

6   8
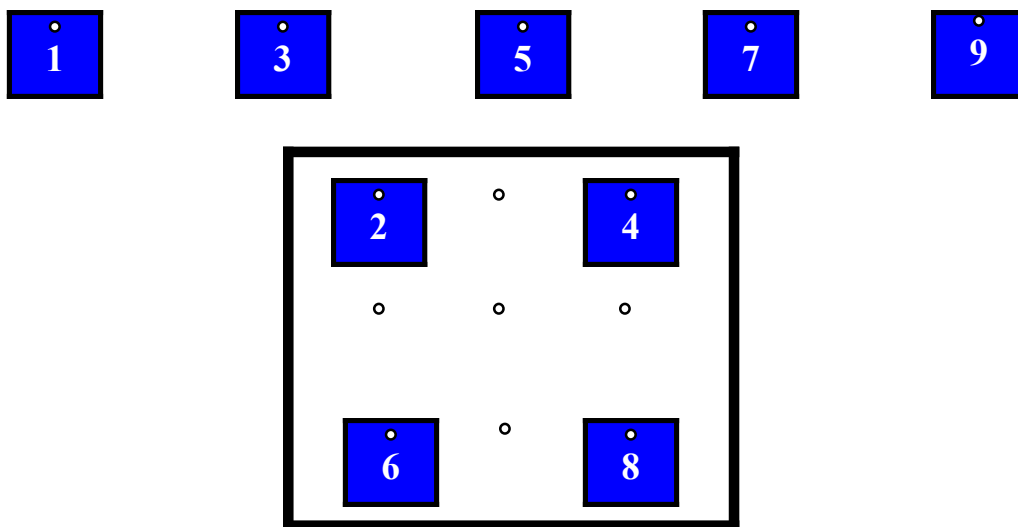
This was very easy for Eliza because Mrs. Matisse told her how to skip count and she was very good at following instructions. With the numbers that were remaining, Mrs. Matisse taught the children to skip count with odd numbers. She then gave Chelsea instructions to but the odd numbers on the remaining pegs starting from the bottom middle and then going to the right most peg in the middle row. When Chelsea finished, the board looked like this:
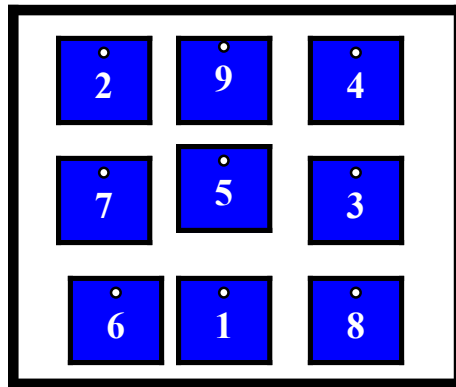
What the students did not know was that if you added the numbers in groups of three (rows, columns and two major diagonals), they summed to 15. The purpose of this lesson was to prepare them to add two to a number.

These simple little tricks were going to make it easy for the students to learn to add when they went to the first grade. Counting forwards and backwards would allow them to add 1 and 9 to a number, Skip counting forward and backwards would allow them to add 2 and 8 to a number. Learning them to think of 6 and 7 as 5+1 and 5+2 would help them adding 5, 6, and 7 to a number. They were going to have to wait for the first grade before they could add 3 and 4 to a number.

Soon it was the end of the semester and Chelsea, Eliza, and there classmates were among the many children who loved math, were excited about it, and were very good at addition. Mrs. Matisse made it so much fun for them.

Soon summer vacation was over, and the children returned to the second grade where Mrs. Oates was going to be their teacher. For the first two weeks of school, Mrs. Oates reviewed all that Mrs. Matisse had taught them. Then she had them memorize that 5-2=3 and that 5-1=4. And that 3+4=7. Then she wrote the following on the board:

```
 1   2   3   4   5   6   7   8   9
+3  +3  +3  +3  +3  +3  +3  +3  +3
 4   5       7          10
```
These were the only ones to which the students knew the answers

So Mrs. Oates wrote the following:

$$
\begin{array}{cc}
3 & \textbf{3} \\
\underline{+3} & \underline{+\textbf{4}-1} \\
6 = & 7 \quad -1
\end{array}
\qquad\qquad
\begin{array}{cc}
4 & \textbf{3}+1 \\
\underline{+4} & \underline{+\textbf{4}} \\
8 = & 7 +1
\end{array}
$$

The students realized that by memorizing 3+4, they could just add or subtract one from the answer to get another math fact.

Then she wrote:

$$
\begin{array}{cc}
3 & \textbf{4}-1 \\
\underline{+6} & \underline{+\textbf{6}} \\
9 \;=\; & 10 - 1
\end{array}
\qquad\qquad
\begin{array}{cc}
4 & \textbf{3}+1 \\
\underline{+7} & \underline{+\textbf{7}} \\
11 = & 10 +1
\end{array}
$$

In this example Mrs. Oates made use of the tens complement facts that Mrs. Matisse had taught them.

Next she wrote:

$$
\begin{array}{cc}
3 & \textbf{5}-2 \\
\underline{+5} & \underline{+\textbf{5}} \\
8 \;=\; & 10 -2
\end{array}
$$

When the students went back to the adding 3 to a number they now knew the following:

$$
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\underline{+3} & \underline{+3} & \underline{+3} & \underline{+3} & \underline{+3} & \underline{+3} & \underline{+3} & \underline{+3} & \underline{+3} \\
4 & 5 & 6 & 7 & 8 & 9 & 10 & &
\end{array}
$$
These were the only ones to which the students knew the answers

Mrs. Oates told them this approach to learning math was the way professional mathematicians approach solving problems. Now they could see that they had to learn tricks for adding and 9 to a number. Then she gave them a set of problems for adding 4:

$$
\begin{array}{ccccccccc}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
\underline{+4} & \underline{+4} & \underline{+4} & \underline{+4} & \underline{+4} & \underline{+4} & \underline{+4} & \underline{+4} & \underline{+4} \\
5 & 6 & 7 & 8 & 9 & 10 & 11 & &
\end{array}
$$
These were the only ones to which the students knew the answers

Again the students saw that they needed to learn a trick for adding 8 or 9 to a

number.   All eyes were on the Math Magician. He reminded them that they had been taught to count backwards and to skip count backwards. Now all they had to do was to apply that knowledge. Mrs. Oates gave them the following numbers and told them to count back one to get an answer:

9 2 8 3 7 4 6 5 Mrs. Oates wrote this
8 1 7 2 6 3 5 4   The students wrote this

Now she asked them to concatenate (put next to) a one in front of each number:
9   2   8   3   7   4   6   5 Mrs. Oates wrote this
18 11   17 12 16 13 15 14   The students wrote this

Now Mrs. Oates wrote the following and ask them to count back one and concatenate a one in front of the answer:

9   2   8   3   7   4   6   5 Mrs. Oates wrote this
+9 +9 +9 +9 +9   +9 +9 +9
18 11   17 12   16 13 15 14   The students wrote this

Now Mrs. Oates was going to let the children teach them selves how to add eight to a number. She gave them the following:

9   2   8   3   7   4   6   5 Mrs. Oates wrote this
+8 +8 +8 +8 +8   +8 +8 +8

The students now had to figure this out for themselves. Chelsea and Eliza asked Mrs. Oates if she would summarize the rules for there seemed to be so much that she had taught them.   Mrs. Oates wrote the following:

9+1=10: The nine rule as add ten to a number and count back 1.
8+2=10: The eight rule is skip count back and add 10
3+7=10:   4+7=11 (4=3+1)
4+6=10:   3+6=9 (3=4-1)
4=5-1:   5+4=10-1=9
3=5-2   5+3=10-2=8
3+4=7:   3+3=6 (4-1)   4+4=8 (3+1)
6=5+1:   6+7=5+1+5+2=10+3=13
         6+6=5+1+5+1=10+2=12

6+5=5+1+5=10+1=11

7=5+2   7+7=5+2+5+2=10+4=14

7+5+7+2+5=10+2=12

Because eight rules seemed a lot, Chelsea and Eliza asked if there were a small set of rule. Mrs. Oates wrote the following.

Addition is about making tens
 1. For the 9 & 8 rule we count back 1 or 2 and add ten to our number.
 2. We got more help with ten complements with3+7=10 and 6+4=10
 3. We think of 3=5-2, 4=5-1, 6=5+1, and 7=5+2 so that we can make tens with
    two fives
 4. We needed a little outside help with 3+4=7

Mrs. Oates told the class that Chelsea and Eliza asked very good questions because that gave her an opportunity to teach her class how to learn and study. To help her students to practice and learn their facts she gave them the following quiz several times a week:

Daily Quiz

Name:_____   Date: _____

2	3	4	5	7	8	9
+10	+10	+10	+10	+10	+10	+10

Time   10s

9	2	8	3	7	4	6	5
+2	+2	+2	+2	+2	+2	+2	+2

Time   2s

9	2	8	3	7	4	6	5
+9	+9	+9	+9	+9	+9	+9	+9

Time   9s

9	2	8	3	7	4	6	5
+8	+8	+8	+8	+8	+8	+8	+8

Time   8s

7	5	7	6	6	7	3	5
+6	+5	+7	+5	+6	+5	+5	+4

Time   5s

3	4	3	4	3	3	4
+3	+4	+4	+6	+6	+7	+7

Time   rest

This was another exciting year for Chelsea and Eliza. They had learned so much from Mrs. Oates. Mrs. Oates reviewed everything that Mrs. Matisse had taught them. But she went one big step further in that Mrs. Oates taught them to apply what they learned and she allowed them to teach themselves by following Mrs. Oates example.

Another summer vacation, and a new teacher named Mrs. Lynden. As with Mrs. Oates, Mrs. Lynden spent two weeks reviewing what Mrs. Matisse and Mrs. Oates had taught them. Mrs. Lynden was pleasantly surprised on how much that they had remembered what these two teachers had taught them. Mrs. Lynden noticed that the cognizant skills of these students was much higher than her other students. After 2 months of school, 6 of her 21 students mastered their addition facts. The Math Magician was available to teach her some measuring techniques that would help her track and help her students more effectively.

Mrs. Lynden would have the students practice these facts many times:

$$\begin{array}{cccccccc} 9 & 5 & 8 & 5 & 7 & 3 & 6 & 5 \\ +1 & +1 & +2 & +2 & +3 & +4 & +4 & +5 \end{array}$$

The secret to Mrs. Lynden's success in teaching the class was her record keeping and analysis. She would give the quiz on the previous page once a week. The students would time themselves in seconds with each line using the clock she displayed on the board. Then Mrs. Lynden would put the following for each student on a worksheet:

Name	10s	2s	9s	8s	5s	rest	sum	#correct	errors
Student 1	10	14	9	18	12	12	75	44	8+2,7

She would record each of the times, the number correct answers and the errors. She would use the sum function on the worksheet to find the total time for all the lines. In this case the student took 18 seconds for the 8s and the total time was 75 seconds. The student made two errors 8+2 and 8+7. Then Mrs. Lynden would order all the students by the sum. The goal was for the top students to break 60 seconds, the next level to break 90 seconds and the rest to break 120. She would know where the students were having difficulties and could teach to those difficulties. This student needed practice with the eights and in particular with 8+2 and 8+7. In general she would see each student improve from week to week. Chelsea and Eliza where excited when the finished the quiz in less than 60 sec-

onds and got them all correct. They would cheer for their classmates when they achieve the goal also.