Simply Fractions

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Pick a number from 1 to 15

1	3	5	7
9	11	13	15

2	3	6	7
10	11	14	15

4	5	6	7
12	13	14	15

8	9	10	11
12	13	14	15

2	2	2	2	-2	-2	-2	-2
+4	-4	+-4	- -4	+4	-4	+-4	- -4
6	-2	-2	6	2	-6	-6	2

2-4=-(4-2)

-2+4=(4-2)

- - = + =++

+- = - =-+

2	2	2	2	1/2	1/2	1/2	1/2
x4	/4	x1/4	/1/4	x4	/4	x1/4	/1/4

2 / 4=2/4=1/2 1/2x4=4/2=2

/ / = x = x x

x / = / = / x ?

1/3+1/3+1/3 = 3x1/3=1 definition of a fraction

-3=0 -3 1/3=1 / 3 = 1÷3 (a)

3=0 - - 3 3 = 1 / 1/3 //=x (b)

0=1 - = / +=x -3=1/3

3+0=3	identity	3X1=3
3+0-2=3-2	sub/div	3x1 / 4=3/4
3+(0-2)=3-2	assoc	3x(1 /4)=3/4
3+-2=3-2	transitive	3x1/4=3/4 (a)

3+0=3	identity	3x1=3
3+0- - 2=3- -2	sub/div	3x1 / 1 /4=3/ 1/4
3+(0- -2)=3- -2	assoc	3x(1/ 1/4)=3 / 1/4
3+2 =3- -2	transitive	3x4 =3 / 1/4 (b)

1/3+1/3+1/3 = 3x1/3=1 definition of a fraction

-3=0 -3 1/3=1 / 3 = 1—3 (a)

3=0 - - 3 3 = 1 / 1/3 (b)

-3+3+-5+5=0+0=0	**def**	**1/3x3x1/5x5=1x1=1**
-3+-5+3+5=0	**comm**	**1/3x1/5x3x5=1**
(-3+-5)+(3+5)=0	**assoc**	**(1/3x1/5)x(3x5)=1**
-3+-5=0-(3+5)	**sub/div**	**1/3x1/5=1 / (3x5)**
-3+-5=-(3+5)	**trans**	**1/3x1/5=1/(3x5)**

Something to think about:

1/3=/3

3X1/3=1=3x/3 x/=/

3/ 1/4=3//4=4x3 //=x corr

What is a Fraction?

Division is reverse multiplication:

3x2=6 3x? =2

3 =6 /2 ?= 2/3 Note: 2/3 = 2 ÷ 3

2=6/3

This is multiplication: 3x5=5+5+5

Now let's make a discovery!

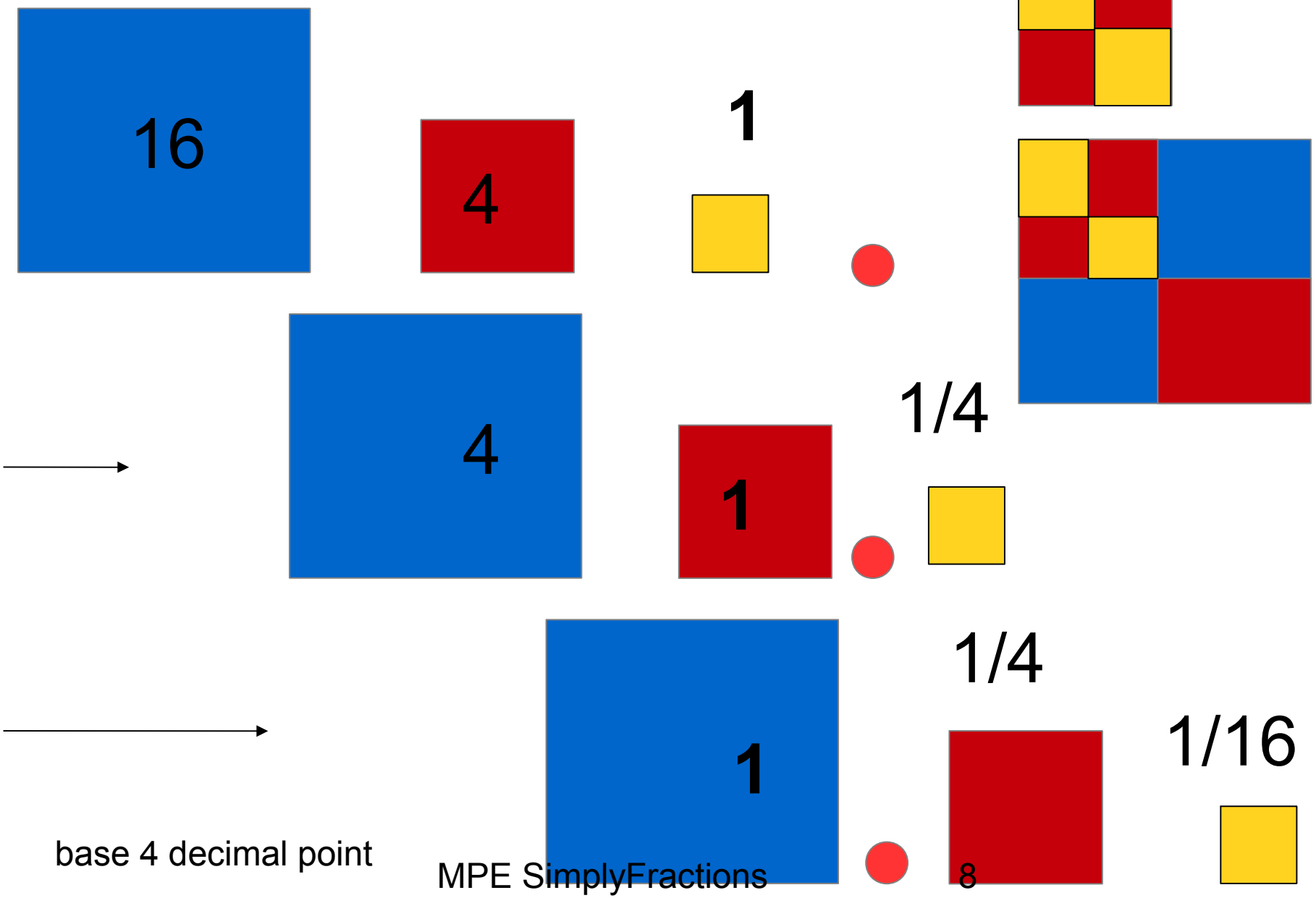
5x1/5 = 1/5 +1/5+1/5+1/5+1/5=1

A (unary) fraction is defined as:

n x 1/n =1 1/n = /n (my notation)

A (unary) fraction is defined such that when it is multiplied by its inverse (the denominator), we get the identity element for multiplication.

Fraction Point



base 4 decimal point

Classic mistakes

A month is not 1/12 of a year, because the multiplication assumes **all the parts are the same.** Jan=31 days April=30 days

A week is not 1/52 of a year 1 year =365 days 7x52=364
The product must be 1.

Cut a dollar into 100 pieces. One piece is not a cent, it is a piece of confetti. A **part of a whole** is **not the correct definition** for a fraction.

What is a proper fraction?

Basic Math properties:

- **Associative (grouping)**
- **Commutative (ordering)**
- **Identity (0 fror +, 1 for x)**
- **Distributive (2+3)x7=2x7+3x7**
- **Transitive =**

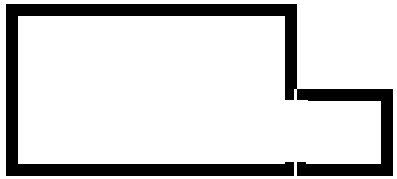
	3 x 1/3 = 1	defintion
(A)	1/3 = 1 / 3	division
	3 x 1 = 3	identity
	3x1 / 7=3 / 7	division
	3x(1 / 7)= 3 / 7	association
	3 x (1/7) = 3 /7	transitive (A)

Notation is very important (The little things count).

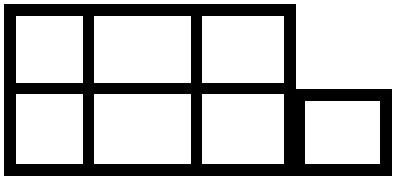
Dissecting a Proper Fraction

3/7 usually read as three sevenths
3/7 3 divided b y 7
3/7= 3 x 1/7 **3 x /7 my notation**
3/7= 1/7 x 3 commutative property **/7 x 3 my notation**

Just think of 1/7 (/7) as just another number element as one would think of an integer as a number element

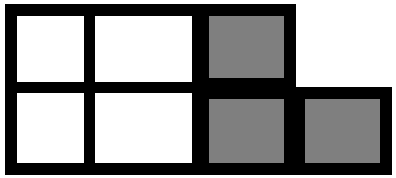


A whole (1)



7 (equal) parts=whole

1/7



3 x 1/7 = 3/7

Why do we add just the numerators when adding fractions (and not add the denominators too).

1/7+ 2/7 + 3/7		
=1 x1/7 + 2x1/7 + 3x1/7	transitive (notation)	
1x/7+2x/7+3x/7		
=(1+2+3)x1/7	distributive	(1+2+3)x/7
=6 x 1/7	adding	6x/7
=6/7	transitive (notation)	

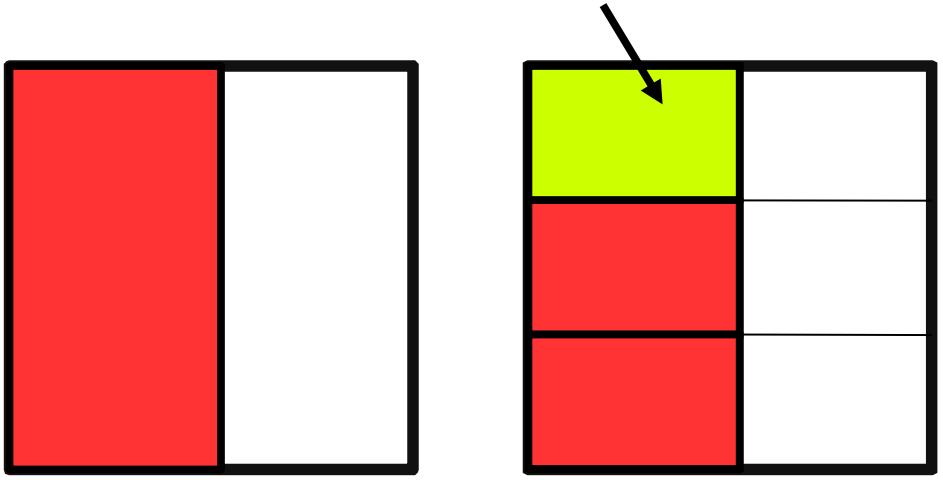
Answer: **Because of the distributive property.**

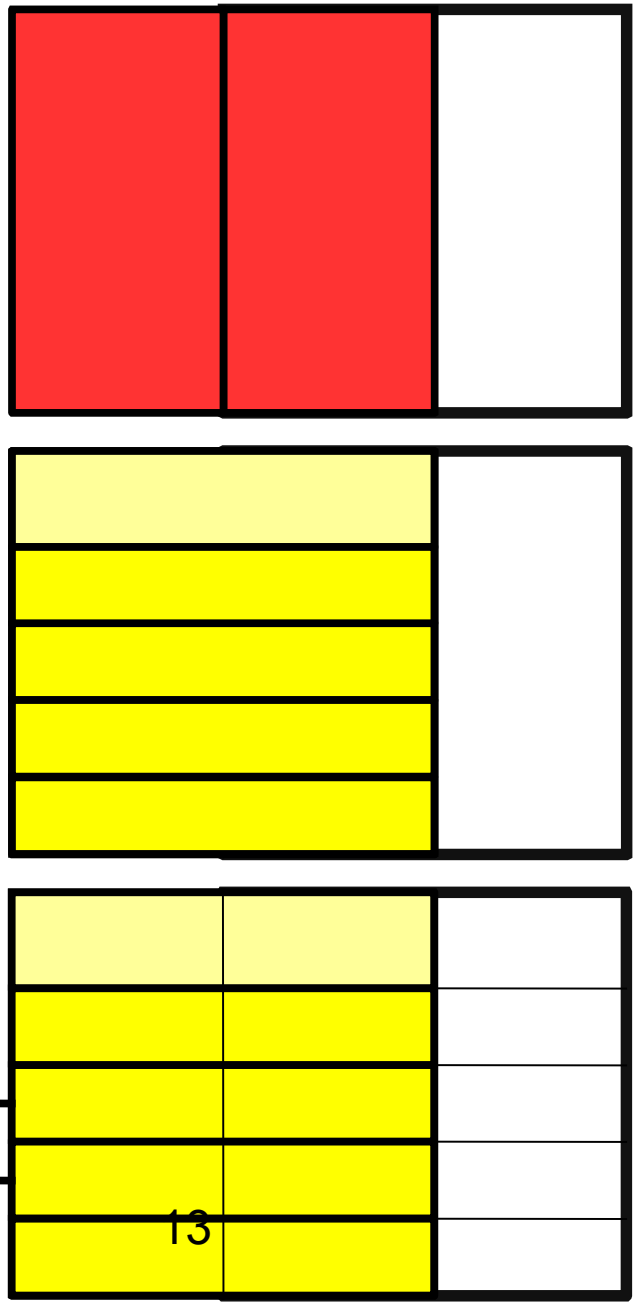
That is why the denominators must be the same!

Question, use manipulatives to validate (verify)

Multiplying Fractions

1/2 x1/3= 1/(2x3)=1/6





2/3 x 4/5 =2x 1/3 X 4 x 1/5

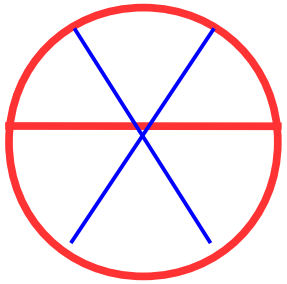
=2x4 x1/3 x1/5

=8 x 1/15=8/15

Adding Fractions with different **denominators**

1x6=6 1=6/6 also 6=6/1

1/2= 1x 1/2 = 3/3 x 1/2= 3x1 /2x3= 3/6

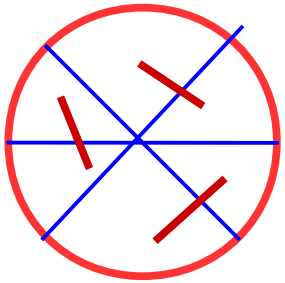


1/2+1/3= 1x 1/ 2 + 1 x 1/3

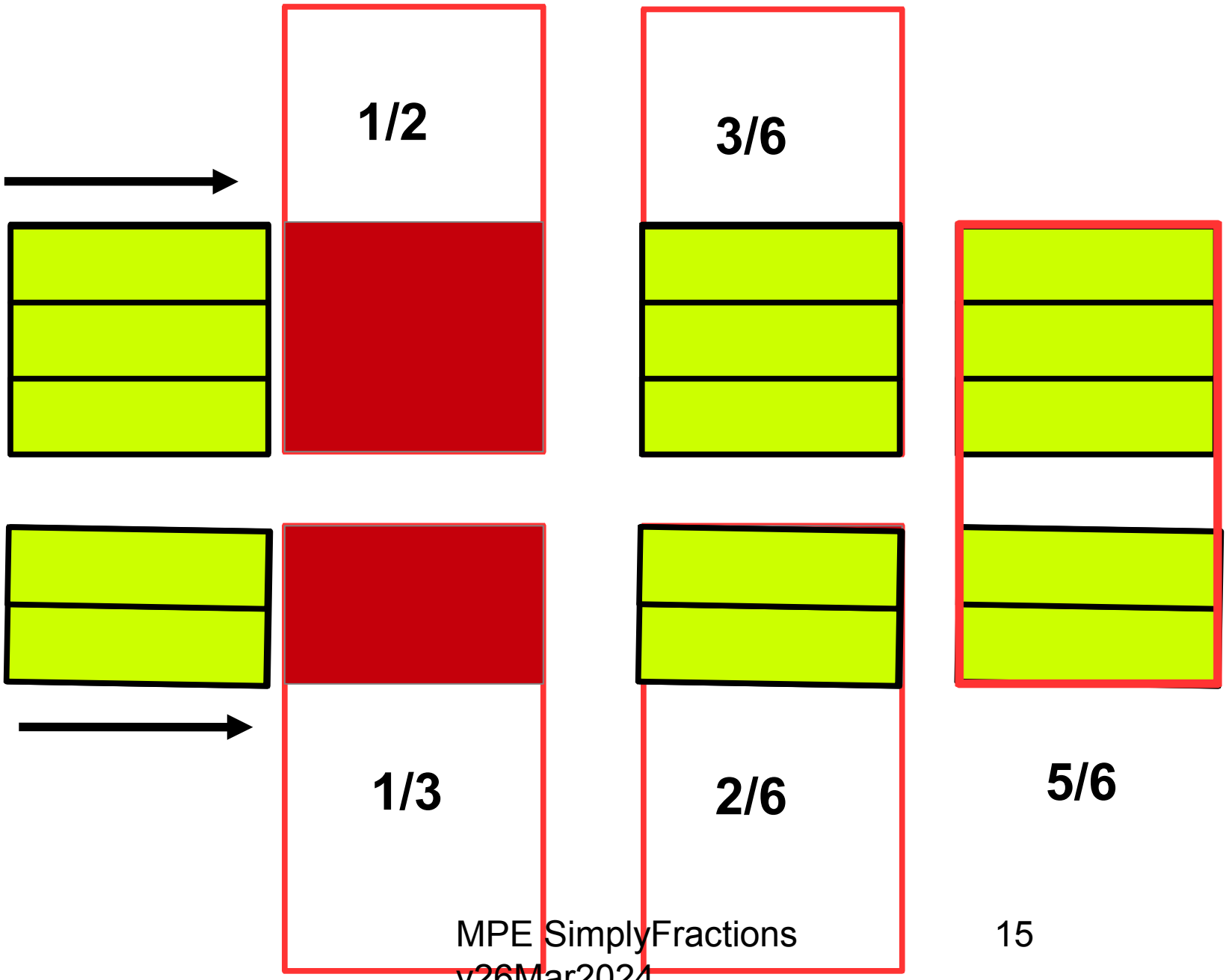
= 3/3 x 1/2 + 2/2 x1/3

= 3x1/(3x2) + 2x1/(2x3)

=3/6 +2/6= 5/6



Adding fractions with different denominators



Dividing fractions

6/5 x 2/3= 12/15

5/6 x6/5 x2/3 = 5/6 x 12/15

5X6/(5x6)x2/3 =12/15 x5/6

30/30 x2/3 =12/15 x5/6

1X2/3 = 12/15 x5/6

2/3 =

2/3= 12/15 / 6/5

inverted

12/15 x 5/6

12/15= 4x3/(5x3)= 4/5 x3/3=4/5 x 1=4/5

4/5 / 5/6= 4x5/(5x6)=4/6 x 5/5 =4/6

4/6 = 2x2 /(2x3)=2/2 x 2/3 =1 x2/3 =2/3

Improper fractions

$$4\ 2/3 = 4 + 2/3 = 3/3 \times 4 + 2/3$$
$$= (4 \times 3)/3 + 2/3$$
$$= (4 \times 3 + 2)/3$$
$$= 14/3$$

$$14/3 = (4X3+2)/3 = 4x3/3 + 2/3$$
$$= 4/1\ x3/3 + 2/3$$
$$= 4 + 2/3$$
$$= 4\ 2/3$$

Reducing Fractions

375/1000= 3x5x5x5/(2x2x2x5x5x5)
=3/8 x5x5x5/(5x5x5)=3/8 x1x1x1=3/8

75x5/(5x200)=75/200 x(5/5)=75x200
=5x15/(5x40)=15/40x5/5)=15/40
=3x5/(8x5)=3/8 x 5/5=3/8

Summary

Adding fractions with same denominators
Multplying fractions
Integer as a fraction
1 as a fraction
Adding fractions different denominators
Reducing fractions
Mixed numbers

The Big Focus (Multiplication & Division)

Remember 3x1/3=1 and 1/3 = 1 / 3 (a)
Note power of associative property:
35 x 1/35 =1=(3x5) x 1/(3x5) so 1/(3x5)= 1 / (3x5)
Let's begin:

3x1/3 x 5 x 1/5 =1x1=1	**definition**	**3x1/3 x 5 x 1/5 =1x1=1**
3x5 x 1/3 x 1/5=1	**commutative**	**5x 1/3 x 3x 1/5=1**
(3x5) x (1/3x1/5) =1	**associative**	**(5x1/3) x (3x1/5)=1**
1/3 x1/5 = 1 / (3x5)	**division**	**3x 1/5 = 1 / (5x1/3)**
1/3 x 1/5 = 1/(3x5)	**(a) transitive**	**3/5 = 1 / 5/3**
multiply numerators		**invert fraction in division**

Continuing:

(2x7) x 1/3 x 1/5 =(2x7) x 1/(3x5)	**multiplying**
2 x 1/3 x 7 x 1/5 =(2x7) x (1/(3x5)	**commutative**
2/3 x 7/5 = =(2x7)/(3x5)	**transitive (notation)**

Adding fractions of different denominators (The importance of 1)

Note: 1=7x 1/7 = 7/7 notation

Just make the denominators the same

2/5 + 3/7

= 1x2/5 + 1x3/7 identity

=7/7 x 2/5 + 5/5 x 3/7 transitive

= (7x2)/(7x5)+(5x3)/(5x7) multiply fractions

=((7x2)+(5x3))/(5x7) adding fractions

=(14+15)/35 =29/35

Reducing fractions(Prime Numbers)

60/72=(2x2x3x5)/(2x2x2x3x3) Transitive (factoring)

=2/2 x 2/2 x 3/3 x 5/(2x3) multiply fractions

=1x1x1x5/6=5/6 transitive

Mixed numbers and improper fractions

Note: $3 \times 1 = 3$ identity

$3 = 3/1$ division

$3\ 3/7 = 3 + 3/7$ notation

$= 3/1 + 3/7$ transitive

$= 7/7 \times 3/1 + 3/7$ identity

$= (7 \times 3)/(7 \times 1) + 3/7$ multiplying

$= ((7 \times 3) + 3)/7$ adding

$= (21+3)/7 = 24/7$ transitive

Applying what we learned

3 2/7 x 7 3/7 = ?

The old way

23/7 x 52/7 = 23x52 /49

3+2/7	52	24 r 20
x 7+3/7	x23	49)1196
21+2	156	-98
(1+2/7) +6/49	104	216
24 20/49	1196	-196
		20

Time consuming and prone to mistakes

Discovery

Fractions and modular arithmetic

On a clock 12=2x12=3x12=4x12=…

1/4 x (8 8+12 8+24 8+36)
= 2 5 8 11 4 answers

Check:

4x2 =8 4x5 =20=(20-12)=8
4x8 =32=(32-24)=8 4x11=44=(44-36)=8

Fractions and exponents

$$16\text{^}1/4 = (16 \times e^{2n\pi i})^{1/4} = 2 \times (e^{0}_{n=0} \; e^{\pi i/2}_{n=1} \; e^{\pi i}_{n=2} \; e^{3\pi i/2}_{n=3})$$
$$= 2 \quad 2i \quad -2 \quad -2i \qquad \text{4 answers}$$

Summary

Importance of the basic postulates of arithmetic
A set of rules based upon observation

Knowing definitions is important

Proving the rules lets us understand what we are doing and makes it easier to retain what we learned

Manipulatives should be used for validation

Understanding leads to discovery (thinking)

n x 1/n =1=n/n defintion **n/m = n x 1/m**
n/m x p/q=(n x p)/(m x q) **1 / (n/m) = m/n**
n/m + p/q=((n x q) +(m x p))/(mxq)

Sample Problems

1/11 + 5/11 + 4/11 =

3/7 + 2/5=

2/9 x 5/7 =

6/72 =

Convert to mixed and improper fractions

23/5 =

3 2/5 =

Do two ways

3 2/7 x 7 3/7 =

Reduce

26/65

Division

4 / 1/2= 4 x 1/ 2=

1/4 / 1/2= 1/4 x 1/2=

1/ 2 / 4= 2 / 4